My Maths Cloud

Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2K: Decision Mathematics 1 and
Decision Mathematics 2

Sample Assessment Material for first teaching September 2017 **Time: 1 hour 40 minutes**	Paper Reference **8FM0/2K**

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If a pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.
- Do not return this question paper with the answer book.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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SECTION A

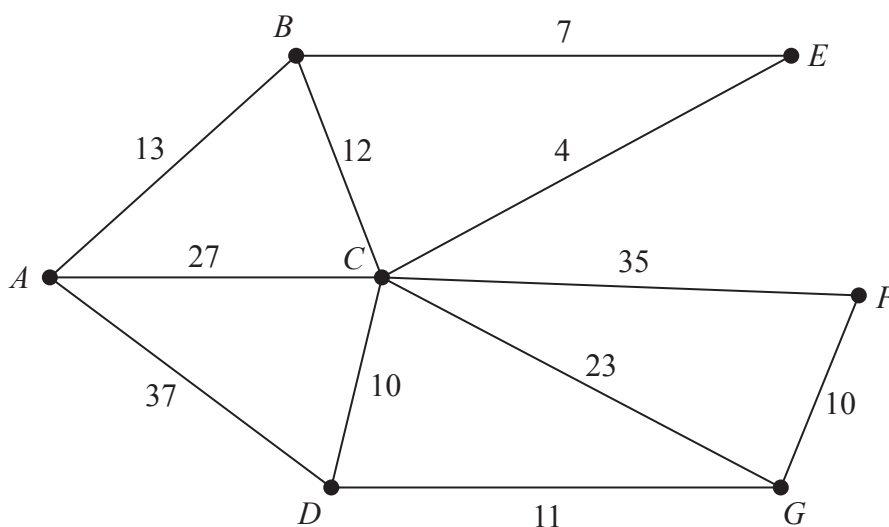**Answer ALL questions. Write your answers in the answer book provided.**

1.

Figure 1

[*The total weight of the network is 189*]

Figure 1 represents a network of pipes in a building. The number on each arc is the length, in metres, of the corresponding pipe.

(a) Use Dijkstra's algorithm to find the shortest path from *A* to *F*. State the path and its length.

(5)

On a particular day, Gabriel needs to check each pipe. A route of minimum length, which traverses each pipe at least once and which starts and finishes at *A*, needs to be found.

(b) Use an appropriate algorithm to find the pipes that will need to be traversed twice. You must make your method and working clear.

(4)

(c) State the minimum length of Gabriel's route.

(1)

A new pipe, *BG*, is added to the network. A route of minimum length that traverses each pipe, including *BG*, needs to be found. The route must start and finish at *A*.

Gabriel works out that the addition of the new pipe increases the length of the route by twice the length of *BG*.

(d) Calculate the length of *BG*. You must show your working.

(2)

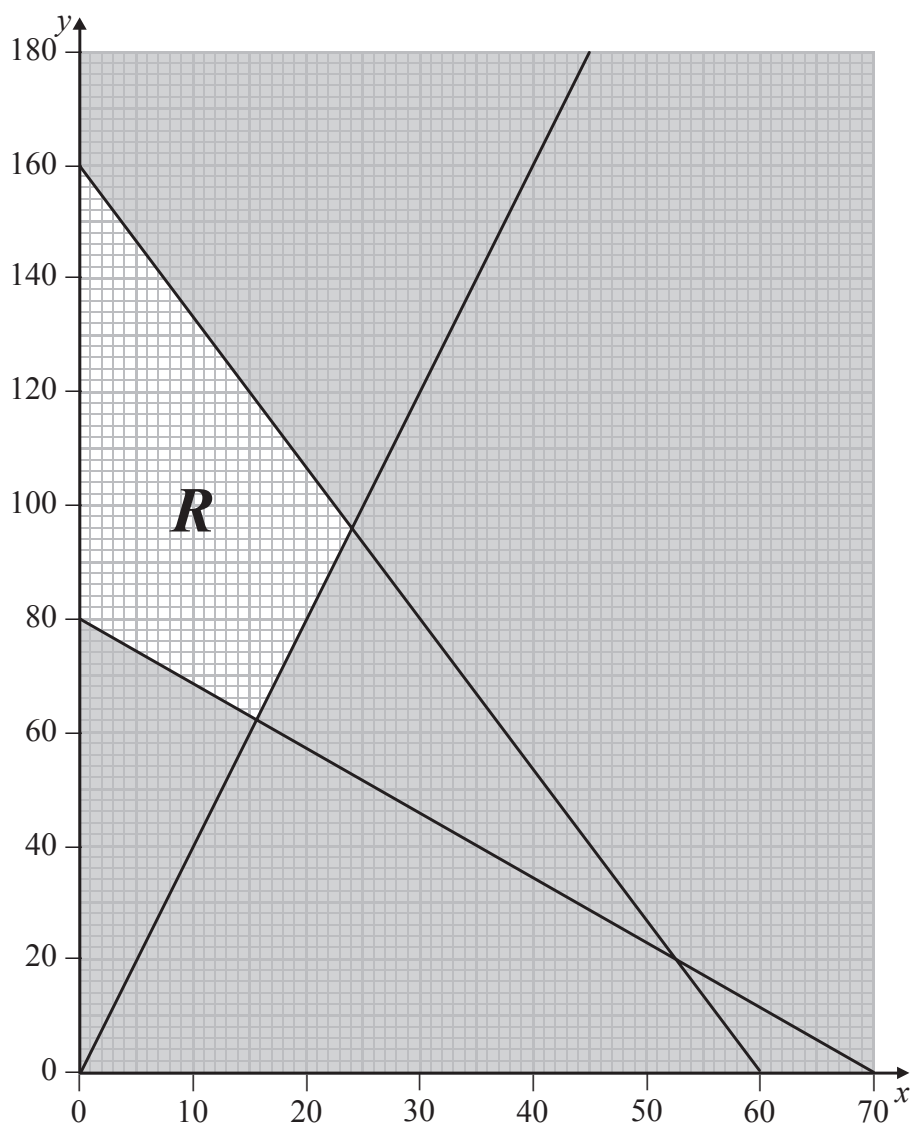**(Total for Question 1 is 12 marks)**

2.

Figure 2

A teacher buys pens and pencils. The number of pens, x, and the number of pencils, y, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leqslant 480$$
$$8x + 7y \geqslant 560$$
$$y \geqslant 4x$$
$$x, y \geqslant 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for Question 2 is 7 marks)

3.

Activity	Time taken (days)	Immediately preceding activities
A	5	–
B	7	–
C	3	–
D	4	A, B
E	4	D
F	2	B
G	4	B
H	5	C, G
I	10	C, G

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time taken in days to complete the activity and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Draw the activity network described in the table, using activity on arc. Your activity network must contain the minimum number of dummies only.

(3)

(b) (i) Show that the project can be completed in 21 days, showing your working.

   (ii) Identify the critical activities.

(4)

(Total for Question 3 is 7 marks)

4. (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5

(1)

A connected graph has exactly 5 nodes and contains 18 arcs. The orders of the 5 nodes are $2^{2x} - 1$, 2^x, $x + 1$, $2^{x+1} - 3$ and $11 - x$.

(b) (i) Calculate x.

   (ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer.

(6)

(c) Draw a graph which satisfies all of the following conditions:

   - The graph has exactly 5 nodes.

   - The nodes have orders 2, 2, 4, 4 and 4

   - The graph is not Eulerian.

(2)

(Total for Question 4 is 9 marks)

5. Jonathan makes two types of information pack for an event, *Standard* and *Value*.

Each *Standard* pack contains 25 posters and 500 flyers.

Each *Value* pack contains 15 posters and 800 flyers.

He must use at least 150 000 flyers.

Between 35% and 65% of the packs must be *Standard* packs.

Posters cost 20p each and flyers cost 4p each.

Jonathan wishes to minimise his costs.

Let *x* and *y* represent the number of *Standard* packs and *Value* packs produced respectively.

Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

You should not attempt to solve the problem.

(Total for Question 5 is 5 marks)

TOTAL FOR SECTION A IS 40 MARKS

Answer ALL questions. Write your answers in the answer book provided.

6. Six workers, A, B, C, D, E and F, are to be assigned to five tasks, P, Q, R, S and T.

   Each worker can be assigned to at most one task and each task must be done by just one worker.

   The time, in minutes, that each worker takes to complete each task is shown in the table below.

	P	Q	R	S	T
A	32	32	35	34	33
B	28	35	31	37	40
C	35	29	33	36	35
D	36	30	34	33	35
E	30	31	29	37	36
F	29	28	32	31	34

   Reducing rows first, use the Hungarian algorithm to obtain an allocation which minimises the total time. You must explain your method and show the table after each stage.

   (Total for Question 6 is 9 marks)

7. In two-dimensional space, lines divide a plane into a number of different regions.
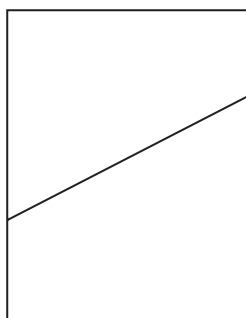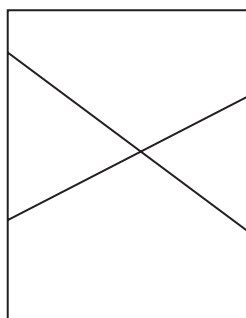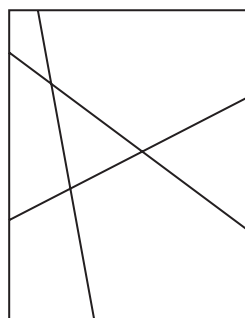

| **Figure 1** | **Figure 2** | **Figure 3** | **Figure 4** |
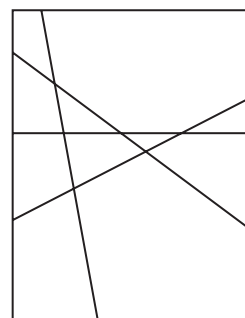
It is known that:

- One line divides a plane into 2 regions, as shown in Figure 1

- Two lines divide a plane into a maximum of 4 regions, as shown in Figure 2

- Three lines divide a plane into a maximum of 7 regions, as shown in Figure 3

- Four lines divide a plane into a maximum of 11 regions, as shown in Figure 4

(a) Complete the table in the answer book to show the maximum number of regions when five, six and seven lines divide a plane.

(1)

(b) Find, in terms of u_n, the recurrence relation for u_{n+1}, the maximum number of regions when a plane is divided by $(n + 1)$ lines where $n \geqslant 1$

(1)

(c) (i) Solve the recurrence relation for u_n

   (ii) Hence determine the maximum number of regions created when 200 lines divide a plane.

(3)

(Total for Question 7 is 5 marks)

8.
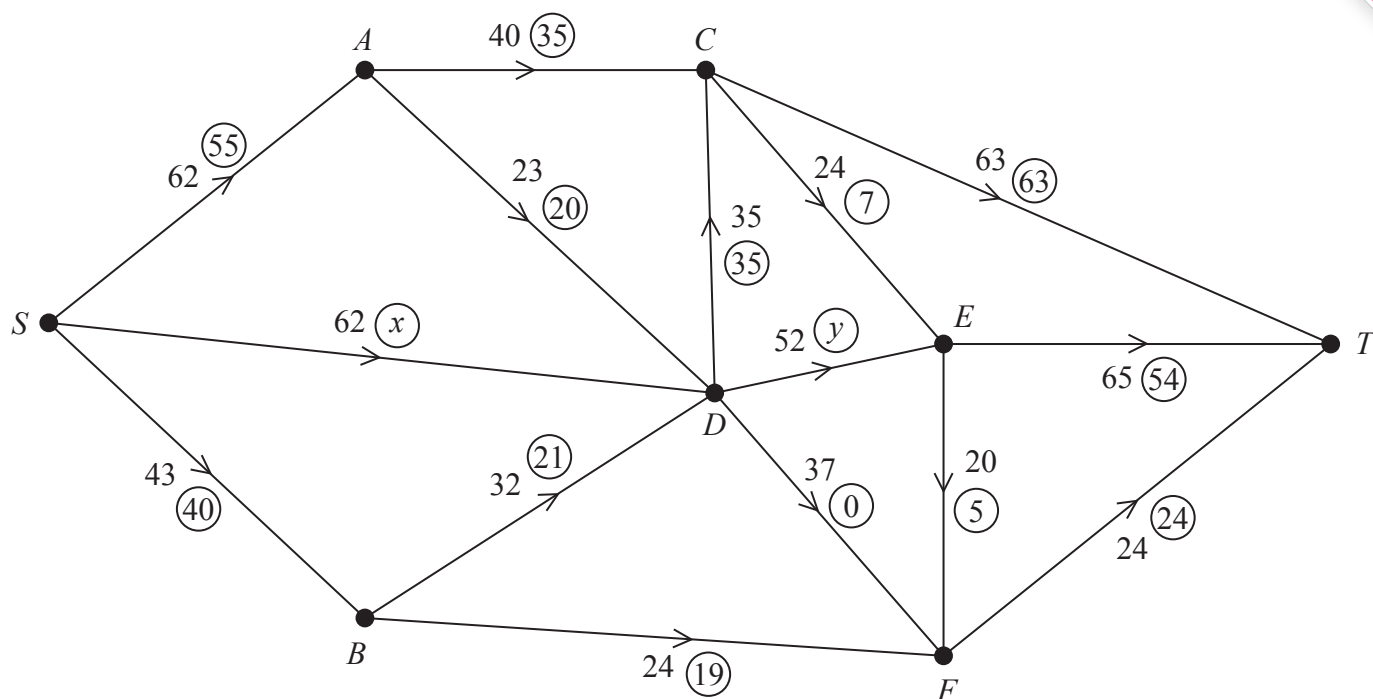
Figure 5

Figure 5 represents a network of corridors in a school. The number on each arc represents the maximum number of students, per minute, that may pass along each corridor at any one time. At 11 am on Friday morning, all students leave the hall (*S*) after assembly and travel to the cybercafé (*T*). The numbers in circles represent the initial flow of students recorded at 11 am one Friday.

(a) State an assumption that has been made about the corridors in order for this situation to be modelled by a directed network.

(1)

(b) Find the value of *x* and the value of *y*, explaining your reasoning.

(3)

Five new students also attend the assembly in the hall the following Friday. They too need to travel to the cybercafé at 11 am. They wish to travel together so that they do not get lost. You may assume that the initial flow of students through the network is the same as that shown in Figure 5 above.

(c) (i) List all the flow augmenting routes from *S* to *T* that increase the flow by at least 5

(ii) State which route the new students should take, giving a reason for your answer.

(3)

(d) Use the answer to part (c) to find a maximum flow pattern for this network and draw it on Diagram 1 in the answer book.

(1)

(e) Prove that the answer to part (d) is optimal.

(3)

The school is intending to increase the number of students it takes but has been informed it cannot do so until it improves the flow of students at peak times. The school can widen corridors to increase their capacity, but can only afford to widen one corridor in the coming term.

(f) State, explaining your reasoning,

   (i) which corridor they should widen,

   (ii) the resulting increase of flow through the network.

<div align="right">(3)</div>

<div align="right">(Total for Question 8 is 14 marks)</div>

9. A two person zero-sum game is represented by the following pay-off matrix for player A.

	B **plays 1**	B **plays 2**	B **plays 3**
A **plays 1**	4	1	2
A **plays 2**	2	4	3

    (a) Verify that there is no stable solution.

               (3)

    (b) (i) Find the best strategy for player A.

        (ii) Find the value of the game to her.

               (9)

(Total for Question 9 is 12 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Write your name here

Surname

Other names

Pearson Edexcel
Level 3 GCE

Centre Number

Candidate Number

Further Mathematics

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Further Mathematics options
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Sample Assessment Material for first teaching September 2017

Paper Reference

8FM0/2K

Answer Book
Do not return the question paper with the answer book.

Total Marks

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SECTION A

1.
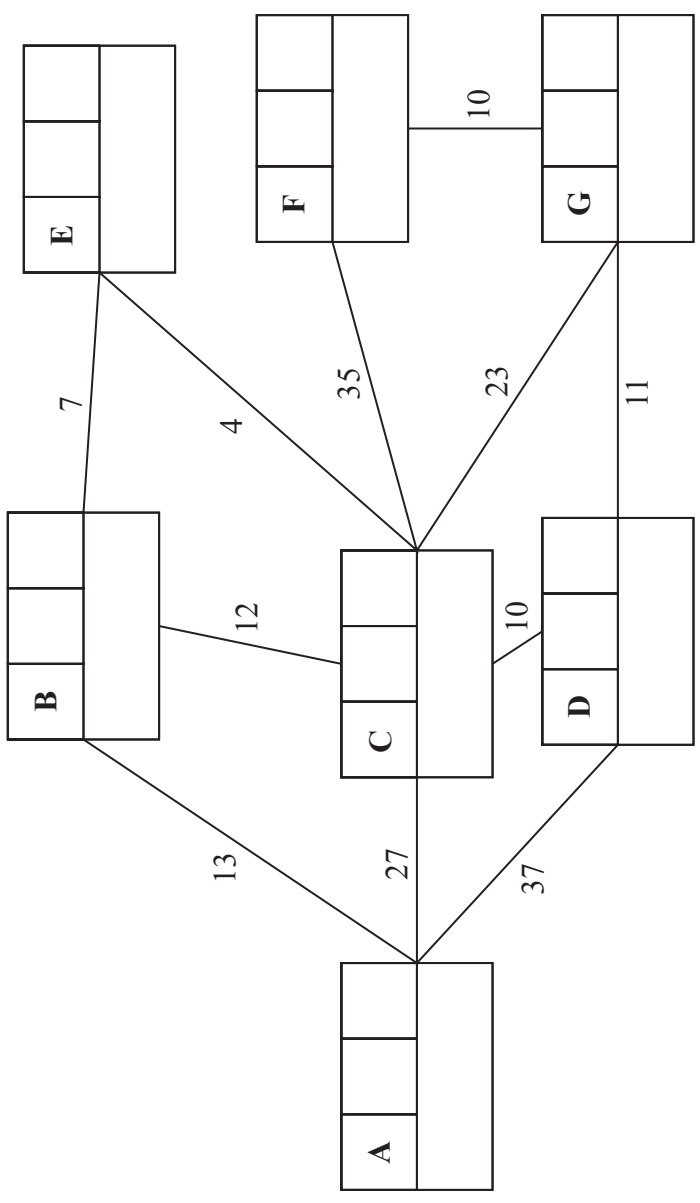
Key:

Vertex	Order of labelling	Final values
	Working value	

Shortest path: _____

Length of shortest path: _____

Question 1 continued

(Total for Question 1 is 12 marks)

2.
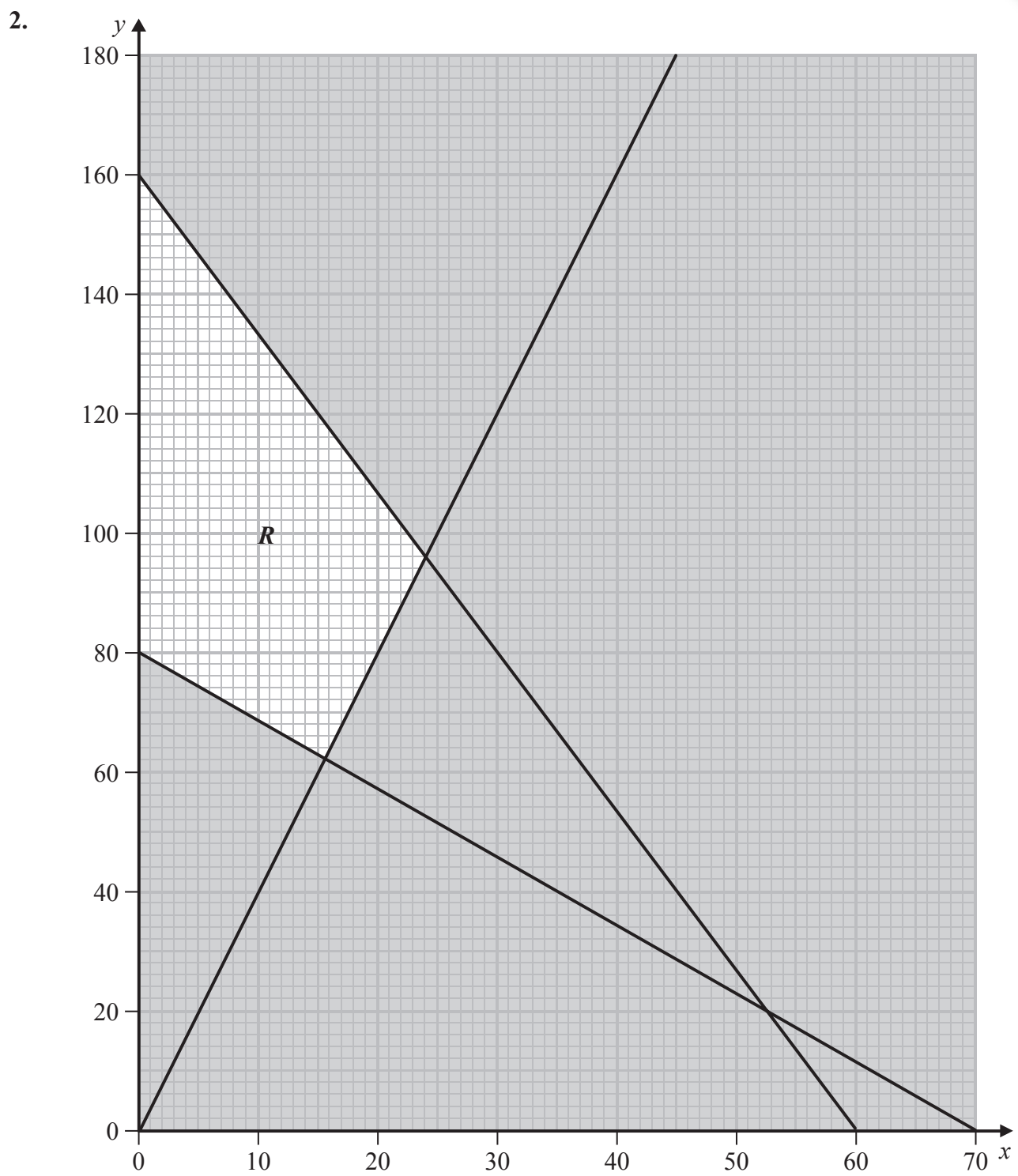
Question 2 continued

(Total for Question 2 is 7 marks)

3. (a) and (b)

(Total for Question 3 is 7 marks)

4.

(Total for Question 4 is 9 marks)

5.

(Total for Question 5 is 5 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

6.

	P	Q	R	S	T
A	32	32	35	34	33
B	28	35	31	37	40
C	35	29	33	36	35
D	36	30	34	33	35
E	30	31	29	37	36
F	29	28	32	31	34

	P	Q	R	S	T	
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	P	Q	R	S	T	
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(Total for Question 6 is 9 marks)

7. (a)

Number of lines	1	2	3	4	5	6	7
Maximum number of regions	2	4	7	11			

(Total for Question 7 is 5 marks)

8.

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(d)
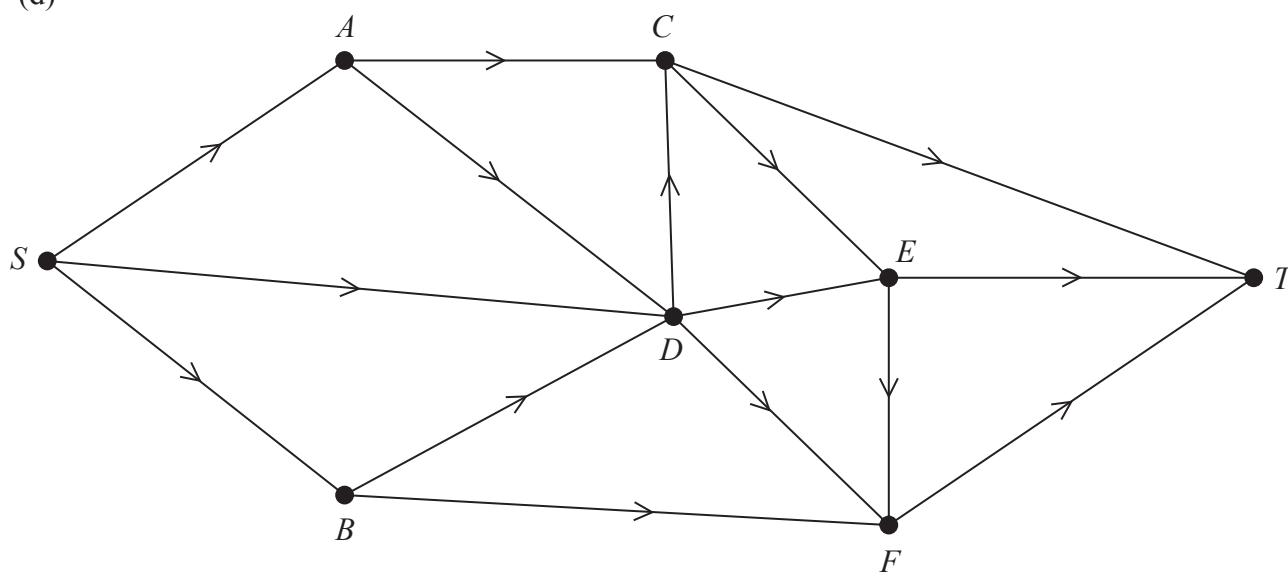
Diagram 1

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(Total for Question 8 is 14 marks)

9.

	B plays 1	*B* plays 2	*B* plays 3
A plays 1	4	1	2
A plays 2	2	4	3

Question 9 continued

(Total for Question 9 is 12 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS

Decision Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)		M1 A1 A1	1.1b 1.1b 1.1b
	Path: ABECDGF	A1	1.1b
	Length: 55 (metres)	A1ft	1.1b
		(5)	
(b)	$AB + DG = 13 + 11 = 24 \leftarrow$	M1	1.1b
	$A(BEC)D + B(ECD)G = 34 + 32 = 66$	A1	1.1b
	$A(BECD)G + B(EC)D = 45 + 21 = 66$	A1	1.1b
	Repeat arcs: AB, DG	A1ft	2.2a
		(4)	
(c)	Length = $189 + 24 = 213$ (metres)	B1ft	1.1b
		(1)	
(d)	$189 + x + 34 = 213 + 2x$	M1	3.1b
	$x = 10$ so BG is 10 m	A1	1.1b
		(2)	
		(12 marks)	

Notes:

(a)
M1: For a larger number replaced by a smaller one in the working values boxes at C, D, F or G
A1: For all values correct (and in correct order) at A, B, C and D
A1: For all values correct (and in correct order) at E, F & G
A1: For the correct path
A1ft: For 55 or ft their final value at F

(b)
M1: For 3 correct pairings of the four odd nodes (A,B, D & G)
A1: At least two pairings and totals correct
A2: All three pairings and totals correct
A3ft: Selecting their shortest pairing, and stating that these arcs should be repeated
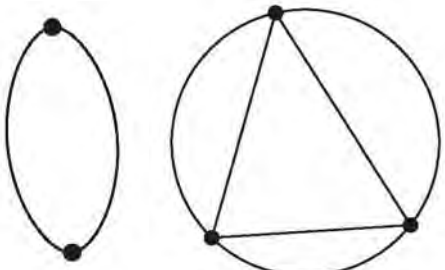
Question 1 notes continued:
(c)
B1ft: For 213 or 189 + their shortest repeat
M1: For translating the information in the question in to an equation involving x, $2x$ and 34
A1: For a correct equation leading to BG = 10 (m)

Question	Scheme	Marks	AOs
2	Objective line drawn or at least two vertices tested	M1	3.1a
	For solving $y = 4x$ and $8x + 7y = 560$ to find the exact co-ordinate of the optimal point, must reach either $x =$ or $y =$	M1	1.1a
	$x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$	A1	1.1b
	Finding at least two points with integer co-ordinates from $(15 \pm 1,\ 63 \pm 2)$	M1	1.1b
	Testing at least two points with integer co-ordinates	M1	1.1b
	$x = 15$ and $y = 63$	A1	2.2a
	So the teacher should buy 15 pens and 63 pencils	A1ft	3.2a
		(7 marks)	

Notes:

M1: Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C

M1: Solving simultaneous equations

A1: cao

M1: Recognition that outcome from this model is non-integer and integer solutions are required – testing two points with integer co-ordinates in at least one of $y \geq 4x$ and $8x + 7y \geq 560$

M1: Testing at least two integer solutions in $y \geq 4x$ or $8x + 7y \geq 560$ and C

A1: cao – deducing from tests which integer solution is both valid and optimal

A1ft: Interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils

Question	Scheme	Marks	AOs
3(a)(b)		M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
		M1 A1	2.1 1.1b
	The number(s) at the end of activity E indicate this project can be completed in 21 days	A1ft	2.2a
	Critical activities: B, G, I	A1	1.1b
		(4)	
		(7 marks)	

Notes:

M1: At least 5 activities and one dummy, one start
A1: A,B,C,D,F,G and first dummy correct
A1: E,H,I correct, second dummy correct and one finish

M1: All boxes completed, number generally increasing L to R (condone one "rogue")
A1: All values cao
A1: Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes)
A1: Critical activities correct

Question	Scheme	Marks	AOs
4(a)	e.g. a graph cannot contain an odd number of odd nodes e.g. number of arcs $= \dfrac{1+3+4+4+5}{2} = 8.5 \notin \mathbb{Z}$	B1	2.4
		(1)	
(b)(i)	$\left(2^{2x}-1\right)+\left(2^{x}\right)+\left(x+1\right)+\left(2^{x+1}-3\right)+\left(11-x\right)=2(18)$	M1	1.1b
	$2^{2x}+3\left(2^{x}\right)-28=0 \Rightarrow x=\ldots$	M1	1.1b
	$\left(2^{x}+7\right)\left(2^{x}-4\right)=0 \Rightarrow x=2$	A1	1.1b
		(3)	
(b)(ii)	The order of the nodes are 9, 15, 3, 4, 5	M1	2.1
	Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes	A1	2.4
		A1	2.2a
		(3)	
(c)		M1	2.5
		A1	2.2a
		(2)	
		(9 marks)	

Notes:

(a)
B1: Explanation referring to need for an even number of odd nodes oe

(b)
M1: Forming an equation involving the orders of the 5 odd nodes and 2(18)

M1: Simplifies to a quadratic in 2^{x} and attempts to solve
A1: 2 cao
M1: Construct an argument involving the order of the 5 nodes
A1: Explanation considering the number of odd nodes
A1: Deduction that therefore it is neither Eulerian nor semi-Eulerian

(c)
M1: Interprets mathematical language to construct a disconnected graph
A1: Deduce a correct graph

Question	Scheme	Marks	AOs
5	Minimise $(C =) 25x + 35y$	B1	3.3
	Subject to: $(500x + 800y \geqslant 150\,000 \Rightarrow)\ 5x + 8y \geqslant 1500$	B1	3.3
	$\dfrac{7}{20}(x+y) \leqslant x \leqslant \dfrac{13}{20}(x+y)$	M1 M1	3.3 3.3
	Which simplifies to $7y \leqslant 13x$ **and** $13y \geqslant 7x$ $x, y \geqslant 0$	A1	1.1b

(5 marks)

Notes:

B1: A correct objective function + minimise
B1: Translate information in to a correct inequality
M1: For translating the information given into the LHS inequality
M1: For translating the information given in to the RHS inequality
A1: Simplifying to the correct inequalities

Decision Mathematics 2 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
6	$\begin{pmatrix} & P & Q & R & S & T & X \\ A & 32 & 32 & 35 & 34 & 33 & 40 \\ B & 28 & 35 & 31 & 37 & 40 & 40 \\ C & 35 & 29 & 33 & 36 & 35 & 40 \\ D & 36 & 30 & 34 & 33 & 35 & 40 \\ E & 30 & 31 & 29 & 37 & 36 & 40 \\ F & 29 & 28 & 32 & 31 & 34 & 40 \end{pmatrix}$	B1	1.1b
	Reducing rows and then columns $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 0 & 0 & 3 & 2 & 1 & 8 \\ B & 0 & 7 & 3 & 9 & 12 & 12 \\ C & 6 & 0 & 4 & 7 & 6 & 11 \\ D & 6 & 0 & 4 & 3 & 5 & 10 \\ E & 1 & 2 & 0 & 8 & 7 & 11 \\ F & 1 & 0 & 4 & 3 & 6 & 12 \end{pmatrix}$ then $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 0 & 0 & 3 & 0 & 0 & 0 \\ B & 0 & 7 & 3 & 7 & 11 & 4 \\ C & 6 & 0 & 4 & 5 & 5 & 3 \\ D & 6 & 0 & 4 & 1 & 4 & 2 \\ E & 1 & 2 & 0 & 6 & 6 & 3 \\ F & 1 & 0 & 4 & 1 & 5 & 4 \end{pmatrix}$	M1 A1	1.1b 1.1b
	e.g. augment by 1 then augment by 1 $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 1 & 1 & 3 & 0 & 0 & 0 \\ B & 0 & 7 & 2 & 6 & 10 & 3 \\ C & 6 & 0 & 3 & 4 & 4 & 2 \\ D & 6 & 0 & 3 & 0 & 4 & 1 \\ E & 2 & 3 & 0 & 6 & 6 & 3 \\ F & 1 & 0 & 3 & 0 & 4 & 3 \end{pmatrix}$ followed by $\begin{pmatrix} & P & Q & R & S & T & X \\ A & 2 & 2 & 3 & 1 & 0 & 0 \\ B & 0 & 7 & 1 & 6 & 9 & 2 \\ C & 6 & 0 & 2 & 4 & 3 & 1 \\ D & 6 & 0 & 2 & 0 & 3 & 0 \\ E & 3 & 4 & 0 & 7 & 6 & 3 \\ F & 1 & 0 & 2 & 0 & 3 & 2 \end{pmatrix}$	M1 A1ft M1 A1ft A1	1.1b 1.1b 1.1b 1.1b 1.1b
	A – T, B – P, C – Q, (D –), E – R, F – S	A1	2.2a
		(9 marks)	

Notes:

B1:   cao – introducing a dummy task and appropriate value
M1:   Simplifying the initial matrix by reducing rows and then columns
A1:   cao
M1:   Develop an improved solution – need to see Double covered +e; one uncovered –e ; and one single covered unchanged. 4 lines to 5 lines needed
A1ft: ft on their previous table – no errors
M1:   Finding the optimal solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 5 lines needed to 6 lines needed (so getting to the optimal table)
A1ft: ft on their previous table – no errors
A1:   cso on final table (so must have scored all previous marks)
A1:   cso – this mark is dependent on all M marks being awarded – to deduce the optimal allocation from the location of zeros in the table

Question	Scheme	Marks	AOs
7(a)	16, 22, 29	B1	1.1b
		(1)	
(b)	$u_{n+1} = u_n + n + 1$	B1	3.3
		(1)	
(c)	As $u_{n+1} = u_n + p(n) \Rightarrow u_n = \lambda n^2 + \mu n + \phi$ and attempt to solve with $n = 1, 2, 3$	M1	1.1b
	$u_n = \dfrac{1}{2} n (n+1) + 1$	A1	1.1b
	20 101 (regions)	A1ft	1.1b
		(3)	
		(5 marks)	

Notes:

(a)
B1: cao

(b)
B1: Translating problem to mathematical model - correct recurrence relation needed

(c)
M1: An attempt to solve the recurrence relation to determine maximum number of regions
A1: cao
A1ft: Substitution of n = 200 into their quadratic u_n expression

Question	Scheme	Marks	AOs
8(a)	Corridors must be one-way	B1	3.4
		(1)	
(b)	e.g. $55 + x + 40 = 63 + 54 + 24$ **or** $7 + y = 54 + 5$	M1	2.4
	$x = 46$ $y = 52$	A1 A1	1.1b 1.1b
		(3)	
(c)	**(i)** SACET ($= 5$) SDFET ($= 5$)	M1 A1	1.1b 1.1b
	(ii) Students must choose SACET, as they cannot travel from F to E	A1	2.2a
		(3)	
(d)		B1	1.1b
		(1)	
(e)	Use of max-flow min-cut theorem	M1	2.1
	Identification of cut through AC, DC, DE, (EF), FT $= 151$ value of flow $= 151$	A1	3.1a
	Therefore it follows that flow is optimal	A1	2.2a
		(3)	
(f)	Consider increasing capacity of arcs in minimum cut	B1	2.1
	Explanation based on a valid argument, such as: • increasing the capacity of any arc other than FT would not increase the flow by more than 1, as total capacity directly in to T is only 152 • increasing the capacity on FT could increase the total flow by 16 (increased flow along SAD, SD and SBD could all be directed through DF to F)	B1	2.4
	Therefore school should choose to widen FT, which could increase the flow through the network by 16	B1	2.2a
		(3)	
		(14 marks)	

Question 8 notes:	
(a)	
B1:	Explanation of assumption to use this model
(b)	
M1:	Either a correct equation, or explanation that flow in = flow out
A1:	cao
A1:	cao
(c)	
M1:	One flow augmenting route found from S to T
A1:	Two correct flow augmenting routes 5+
A1:	Deduce that SACET must be used as students cannot travel from F to E as route is one-way
(d)	
B1:	A consistent flow pattern = 151
(e)	
M1:	Constructing argument based on max-flow min-cut theorem
A1:	Use appropriate process of finding a minimum cut – cut + value correct
A1:	Correct deduction that the flow is maximal
(f)	
B1	Constructing an argument based on arcs in the minimum cut
B1	Detailed explanation as to why choosing anything other than FT does not help
B1	Correct deduction and correct increase in flow of 16

Question	Scheme	Marks	AOs
9(a)	Row minima: 1, 2 max is 2	M1	1.1b
	Column maxima: 4, 4, 3 min is 3	A1	1.1b
	Row maximin (2) $\neq$ Column minimax (3) so not stable	A1	2.4
		(3)	
(b)	Let A play strategy 1 with probability p and strategy 2 with probability 1- p, and using this to get at least one equation in p	M1	3.3
	Then if B plays strategy 1, A's gains are $4p + 2(1- p) = 2p + 2$	A1	1.1b
	If B plays strategy 2, A's gains are $p + 4(1- p) = 4 - 3p$	A1	1.1b
	If B plays strategy 3, A's gains are $2p + 3(1- p) = 3 - p$ 		
	Intersection of $2p + 2$ and $3 - p$ occurs where $p = \frac{1}{3}$	dM1 A1ft	1.1b 1.1b
	Therefore player A should play strategy 1 $\frac{1}{3}$ of the time and play strategy 2 $\frac{2}{3}$ of the time	A1ft	3.2a
	The value of the game to player A is $2\frac{2}{3}$	A1	1.1b
		(9)	
		(12 marks)	

Question 9 notes:
(a)
M1: Finding row minimums and column maximums – condone one error
A1: Row minima and column maxima correct
A1: Explanation involving $2 \neq 3$ and a conclusion
(b)
M1: Translating situation into model by defining variables and constructing at least one equation
A1: One row correct
A1: All three rows correct
M1: Axes correct, at least one line correctly drawn for their expression
A1: Correct graph
M1: Using their probability expectation graph to find the probability by equating their two correct expressions and attempting to solve as far as $p =$
A1ft: ft on their optimal intersection
A1ft: Interpret their value of p in the context of the question – must refer to play, player A
A1: cao